Determination of an Optimum Parametric Combination Using a Surface Roughness Prediction Model for EDM of Al$_2$O$_3$/SiC$_w$/TiC Ceramic Composite

K. M. Patel*; Pulak M. Pandey*; P. Venkateswara Rao*

* Department of Mechanical Engineering, Indian Institute of Technology Delhi, New Delhi, India

Online Publication Date: 01 June 2009
1. INTRODUCTION

Electrical discharge machining (EDM) is one of the most extensively used nonconventional material removal processes. Its unique feature of using thermal energy to machine electrically conductive materials regardless of hardness has been distinctive advantage in the manufacture of mould, die, automotive, aerospace, and surgical components. During EDM, the tool and the workpiece are separated by a small gap, and submerged in dielectric fluid. The discharge energy produces very high temperatures on the surface of the workpiece at the point of the spark. The specimen is subject to a temperature rise up to 20,000°C causing a minute part of the workpiece to be melted and vaporized [1]. The top surface of the workpiece subsequently resolidifies and cools at very high rate. This layer contains high-tensile residual stresses, a network of microcracks and the surface roughness (SR) is also quite high. The SR generated in EDM depends on the process parameters. The amount of heat generated during the process affects the surface characteristics of the workpiece material [2].

Surface finish resulting from conventional machining operation has traditionally received considerable attention, however few attempts were made in the area of unconventional machining. EDM process involves a large number of machining parameters. Hence, it would be difficult to correlate surface finish with machining parameters. Models contribute significantly to a comprehension of the process itself and form the basis for the simulation of the EDM processes and also helps in the selection of optimum cutting parameters. The models thus create a precondition for increased efficiency while ensuring a high product quality at the same time. Some amount of work has been carried out to develop surface finish prediction models in the past but most of them for the EDM of the steels, EDM of ceramic composite composites have received little attention.

Monolithic alumina, because of its excellent physical properties such as high melting temperature, strength, hardness, and corrosion resistance, is one of the most popular ceramic materials used in wear-resistant and structural applications. Conventional sintering and compacting techniques of powder metallurgy, followed by diamond grinding has been used to machine the alumina ceramic components required in real applications [3]. However, the intrinsic brittleness and hardness of alumina make its machining difficult, which results in a higher fabrication cost that prevents wider usage. To overcome drawbacks in machining of ceramics, various remedies have been employed to increase the toughness and machinability simultaneously. The addition of hard, refractory, conductive ceramics such as TiN, TiC, TiB₂, and TiCN in particulate form to Si₃N₄, ZrO₂, and Al₂O₃ has been used as an approach to produce composites with sufficient conductivity for EDM [4]. These electroconductive and toughened ceramics could be shaped by EDM to manufacture complex components, which could significantly enhance the economic competitiveness of ceramic parts.
Recently, the authors found a significant improvement in the mechanical properties of alumina ceramic composites upon incorporation of a sufficient amount of SiC whisker, TiC particles into Al₂O₃ [5, 6]. There is a need to optimize the machining parameters for EDM as it has been developed as an alternative and cost-effective method for manufacturing complex components from alumina ceramic composites [7]. Therefore, in this research work an effort has been made to develop the surface prediction model incorporating discharge current, pulse-on time, duty cycle, and gap voltage as machining parameters for the EDM of SiC whisker-reinforced and TiC particles alumina-based ceramic composite. The developed statistical model for predicting SR is further utilized to find the optimum machining condition using a trust region method.

2. Literature review

Surface quality and dimensional precision will greatly influence parts during their useful life, especially in cases in which the components will be in contact with other elements or materials. Therefore, their study and characterization is extremely important [8]. Surface finish has been an important factor in predicting the performance of any machining operation and influenced by the machining parameters. As EDM is a process involving a large number of parameters, combined influence of significant parameters can only be obtained by statistical modeling. Moreover, surface finish in ceramic components influences mechanical properties such as fracture strength, wear, and lubrication. Therefore, measuring and characterizing surface finish can be considered for predicting electric discharge machining performance.

Bhattacharya et al. [9] developed surface integrity model for the EDM of M2 Die Steel. The mathematical models were developed for SR, recast layer thickness, and surface crack density in terms of peak current and pulse-on duration. The effect of these parameters on the surface integrity was evaluated using response surface methodology (RSM). It is observed from the parametric analysis that at lower peak current and pulse-on duration, i.e., at 2A and 20μs minimum SR can be achieved.

Petropoulos et al. [10] investigated a multiparameter analysis of surface finish imparted to Ck60 steel plates by EDM. The interrelationship between surface texture parameters and process parameters was emphasized. The response considered includes amplitude, spacing, hybrid, as well as random process and fractal parameters. The correlation of these parameters with the machining conditions, namely, pulse current and pulse-on time was investigated. The investigation showed that observed characteristics become more pronounced when intensified machining conditions were used. Process parameters were summarized into empirical equations as function of pulse current and pulse-on time.

Kansal et al. [11] also used RSM to plan and analyze the influence of powder mixed electrical discharge machining (PMEDM) on material removal rate (MRR) and SR. Pulse-on time, duty cycle, peak current, and concentration of the silicon powder added into the dielectric fluid of EDM were chosen as the process parameters. Second order models were constructed along with contour graphs for the selection of important parameters to maximize MRR and minimize SR in EDM of EN-31 tool steel. The recommended optimal process conditions have been verified by conducting confirmation experiments. It is evident that addition of silicon powder into dielectric improves the SR. The analysis of variance revealed that the factor peak current and concentration are the most influential parameters on the SR.

Puertas and Luis [12, 13] developed a series of mathematical models using design of experiments technique combined with multiple regressions. The three essential machining parameters, namely, current intensity, pulse time and duty factor were chosen to explore their impact on experimentally observed responses like SR, MRR, and electrode wear ratio (EWR) in EDM of conductive ceramics, namely, boron carbide and siliconized silicon carbide (SiSiC). The developed mathematical models were helpful in selection of the optimal machining conditions for finishing stages of EDM process. Puertas et al. [14] also attempted modeling of different surface parameters in EDM of SiSiC. The influence of EDM generator parameters and flushing pressure on two spacing roughness parameters, namely, Sm (mean spacing of profile irregularities) and Ppk (peak count or peak density) was studied. The mean spacing between peaks was reported to increase when the current intensity, pulse time, and duty cycle were increased. Increase in all these parameters results into an increase in size of craters produced while EDM and hence mean spacing. It was also reported that Sm decreased with increase in open circuit voltage. It was observed that an increase in open circuit voltage resulted in improved surface finish. This might be due to low thermal conductivity of SiSiC in comparison to metals. The peak density (Ppk) was significantly affected by current density, pulse time, and duty cycle. It decreased with increase in all the process parameters considered. This was due to more energetic pulses which led to craters of greater diameter and depth, and, therefore, it resulted to a lesser number of peaks in the roughness profile of the EDMed surface.

In a recent work, Chiang [15] studied EDM of Al₂O₃ + TiC mixed ceramic using RSM. The models were developed to explain the influences of four machining parameters, namely, discharge current, pulse-on time, duty factor, and open discharge voltage on the performance characteristics like MRR, EWR, and SR. They concluded that discharge current and pulse-on time have statistical significance on the SR.

Different researchers have made attempts to optimize parameters of machining processes from time to time using different optimization models and solution techniques. Jain and Jain [16] used neural network for modeling and optimizing the machining conditions for abrasive flow machining. Jain et al. [17] optimized process parameters of mechanical type advanced machining processes using genetic algorithms giving the details of formulation of optimization models, solution methodology used, and optimization results. Reddy and Rao [18] developed a SR prediction model using response surface methodology for end milling of AISI1045 steel. They also optimized the end milling process using the SR prediction model as the
DETERMINATION OF AN OPTIMUM PARAMETRIC COMBINATION

stable machining surface quality on Al/SiCp composites. Basheer et al. [19] used artificial neural network (ANN) based objective function with help of genetic algorithms. The predicted roughness of machined surfaces based on the ANN model was found to be in good agreement with the unexposed experimental data set. Bacchewar et al. [20] developed SR models for predicting the SR of upward- and downward-facing surfaces in selective laser sintering process. A trust-region-based optimization method was employed to obtain a set of process parameters for obtaining the best surface finish. Considering the above, an attempt has been made in this work to develop a SR model with machining conditions on the basis of the experimental data and then optimize it for the selection of these parameters within the given constrains in the EDM of alumina ceramic composite for achieving better surface finish.

RESPONSE SURFACE METHODOLOGY

Response surface methodology (RSM) is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimise this response. RSM also quantifies relationships among one or more measured responses and the vital input factors. Based on the model of the response, a near optimal point can then be deduced. RSM is often applied in the characterization and optimization of processes [21]. The mathematical models commonly used are represented by

\[ Y = f (I_p, t_e, \eta, U) \pm \varepsilon, \]  

where \( Y \) is the machining response, \( f \) is the response function, \( I_p, t_e, \eta, U \) are EDM process parameters, and \( \varepsilon \) is the error which is normally distributed about the observed response \( Y \) with zero mean. The quadratic equation for a nonlinear relationship between a particular response and four independent parameters can be given as

\[ Y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_1x_2 + b_6x_1x_3 + b_7x_1x_4 + b_8x_2x_3 + b_9x_2x_4 + b_{10}x_3x_4 + b_{11}x_1^2 + b_{12}x_2^2 + b_{13}x_3^2 + b_{14}x_4^2 + \varepsilon \]  

Above equation is used to describe the functional relationship between the machining response, \( Y \) and \( x_1, x_2, x_3, \) and \( x_4, \) coded values of input parameters

<table>
<thead>
<tr>
<th>Table 1.—Levels of the independent factors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sr. no.</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

In order to develop models on the basis of experimental data, careful planning of experimentation is essential. The factors considered for experimentation and analysis of EDM of ceramic composite were discharge current, pulse-on time, duty cycle, and gap voltage.

3. EXPERIMENTAL DETAILS

A well-designed experimental plan can substantially reduce the number of experiments. Therefore, it is essential to have a well-designed set of experiments. The range of values of each factor selected on basis of pilot experimentation was set at five different levels as shown in Table 1. The selected design matrix is a central composite rotatable factorial design consisting of 31 sets of coded conditions. It comprises a full replication of \( 2^4 = 16 \) factorial design plus seven center points and eight star points. All EDM variables at the intermediate level (0) constitute the center points, and the combinations of each of the EDM variables at either of its lowest (−2) level or highest (+2) level with the other three variables at the intermediate levels constitute the star points. Thus the 31 experimental runs allow the estimation of the linear, quadratic, and two-way interactive effects of the process parameters on SR [21]. The coded values of variables used in Eq. (2), were obtained from the following transforming equation:

\[ x_i = \frac{\text{chosen parametric value} - \text{central rank value}}{\text{Incremental parametric value}} \]  

where \( x_i \) is the coded value of discharge current \( (I_p) \), \( x_2 \) is the coded value of pulse-on time \( (t_e) \), \( x_3 \) is the coded value of duty cycle \( (\eta) \), and \( x_4 \) is the coded value of gap voltage \( (U) \).

3.2. Experimentation

An “Electronica leader ZNC” die-sinking EDM machine, with a pulse generator, was used for experimentation. The electrolytic copper rod having diameter 8.5 mm was used as an electrode. In all the experiments, kerosene oil was used as dielectric medium. The electroconductive
Al$_2$O$_3$/SiC$_w$/TiC (AlSiTi) ceramic composite specimens of $20 \times 20$ mm and $5$ mm thickness were used in the present study. AlSiTi ceramic was fabricated by hot pressing at $1700-1800^\circ$C a mixture of $30.9$ vol% SiC whiskers, $23.0$ vol% TiC powder, and balance Al$_2$O$_3$. AlSiTi ceramic composite has density, hardness, fracture toughness, thermal conductivity, and electrical resistivity as $3.90$ g/cm$^3$, $2400$ Hv, $9.6 \pm 0.6$ MPa(m)$^{0.5}$, $63$ W/m$^2$K, and $0.009$ Ωcm, respectively. Machining time for each workpiece in the experiment was $75$ minutes. After EDM, AlSiTi ceramic composite samples were cleaned with acetone. The surface roughness was measured using SR tester (Talysurf 6, Rank Taylor Hobson, England). A traverse length of $5$ mm with a cut-off evaluation length of $0.8$ mm was selected.

4. Results and discussion

Experimental study was conducted to see the effects of discharge current, pulse-on time, duty cycle, and gap voltage on the EDM performance of the AlSiTi ceramic composite. The variation of SR with respect to independent parameters considered for this study is shown in Fig. 1. It is evident from the figure that the pulse-on time influences the SR more predominantly than other factors. It could be due to the fact that any increase in the pulse-on time increases the plasma channel diameter that reduces both energy density and impulsive force. The melted debris cannot be removed completely due to reduction in impulsive force and forms an apparent globule-like recasted layer to degrade SR. Figure 1 shows that an increase in discharge current increases the SR while EDM of AlSiTi composite. The small improvement in

![Figure 1](image.png)

**Figure 1.**—Variation of surface roughness with discharge current at different values of duty cycle and gap voltage.
surface finish has been observed beyond discharge current 6 A. The SR increases first and then decreases with the increase in the duty cycle. It can be seen from Fig. 1 that gap voltage does not affect the quality of the surface appreciably. This shows that there is a scope for finding the optimum value of discharge current, pulse-on time, and duty cycle for obtaining the best quality of the surface. In order to understand EDM process for AlSiTi ceramic composite better, the experimental results were used to develop the mathematical model using RSM. The “Design Expert 7.1.5” software was used for computation of regression constant and exponents. The same software was also used to develop an experimental plan for RSM.

4.1. The Surface Roughness Model

To analyze the experimental data, the checking of goodness of fit of the model is very much required. The model adequacy checking includes test for significance of the regression model, test for significance on model coefficients, and test for lack of fit [21]. For this purpose, analysis of variance (ANOVA) is performed for SR and is given in Table 2.

The fit summary recommends a quadratic model for SR statistically adequate but the lack of fit is significant. The value of \( R^2 \) is over 89%, which means that regression model provides strong correlation between independent variables and the response \( (R_a) \) and gives good explanation of the relationship between the independent variables and the response \( (R_a) \). The calculated \( F \) for the model is 9.51. The computed \( F \) is greater than the \( F_{0.01} \) for a significance level of \( \alpha = 0.01 \). It indicates that the model is adequate for 99% confidence level. The \( P \)-value calculated for the lack of fit terms is less than 0.05. It means the lack of fit is significant, which is not desired.

To fit the quadratic model for SR, the insignificant terms having \( P \)-values more than 0.05 were eliminated by backward elimination process, and the model is reanalyzed for the adequacy. The second order SR model obtained is given as

\[
R_a = 2.85756 + 0.136871X_1 + 0.248329X_2 - 0.0469X_3 - 0.12249X_1^2 - 0.11378X_3^2
\]

(In coded terms) \( (4) \)

\[
R_a = -2.61128 + 1.361771_p + 0.00496t_e + 4.67968\eta - 0.12249t_p^2 - 4.44\eta^2
\]

(In actual factors). \( (5) \)

The ANOVA for the reduced quadratic model for SR is shown in Table 3. The ANOVA of the reduced model indicates that the model is significant as \( R^2 \) statistics is 84.96%, whereas lack of fit is insignificant. Since \( F \) is greater than \( F_{0.01} \), there is a definite relationship between the response variable and independent variables at 99% confidence level. Figure 2 displays the normal probability plot of the residuals for SR. Notice that the residuals are falling on a straight line, which means that the errors are normally distributed. Using the second order model, the SR of the components produced by EDM of AlSiTi ceramic composite can be estimated with reasonable accuracy. Hence, the second order model was considered as an objective function for the optimization using a trust region method.

4.2. Precision of the Surface Roughness Model

Due to experimental error, the estimated parameters and hence the estimated SR are subject to uncertainty. The precision of SR has been estimated by calculating the confidence interval. The confidence interval for the predicted response is \( R_a \pm \Delta R_a \), where \( \Delta R_a \) is given by

\[
\Delta R_a = t_{\alpha/2,DF} \sqrt{V_e}.
\]

### Table 2.—ANOVA table for surface roughness (before elimination).

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F value</th>
<th>P value</th>
<th>( R^2 )</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>3.322966</td>
<td>14</td>
<td>0.237354</td>
<td>9.51</td>
<td>&lt;0.0001</td>
<td>0.8927</td>
<td>( F_{0.01,14,16} = 3.45 )</td>
</tr>
<tr>
<td>Residual error</td>
<td>0.399411</td>
<td>16</td>
<td>0.024963</td>
<td></td>
<td></td>
<td></td>
<td>( F &gt; F_{0.01,14,16} )</td>
</tr>
<tr>
<td>Lack-of-fit</td>
<td>0.349111</td>
<td>10</td>
<td>0.034911</td>
<td>4.16</td>
<td>0.0472</td>
<td></td>
<td>Model is adequate but lack of fit is significant</td>
</tr>
<tr>
<td>Pure error</td>
<td>0.050299</td>
<td>6</td>
<td>0.008383</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3.762377</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3.—ANOVA table for surface roughness (after elimination of insignificant parameters).

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>DF</th>
<th>MS</th>
<th>F value</th>
<th>P value</th>
<th>( R^2 )</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>3.152882</td>
<td>5</td>
<td>0.630576</td>
<td>28.24</td>
<td>&lt;0.0001</td>
<td>0.8496</td>
<td>( F_{0.01,5,25} = 3.86 )</td>
</tr>
<tr>
<td>Residual error</td>
<td>0.558137</td>
<td>25</td>
<td>0.022325</td>
<td></td>
<td></td>
<td></td>
<td>( F &gt; F_{0.01,5,25} )</td>
</tr>
<tr>
<td>Lack-of-fit</td>
<td>0.499924</td>
<td>19</td>
<td>0.026311</td>
<td>2.71</td>
<td>0.1103</td>
<td></td>
<td>Model is adequate Lack of fit is insignificant</td>
</tr>
<tr>
<td>Pure error</td>
<td>0.058212</td>
<td>6</td>
<td>0.009702</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3.711020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Here, \( t \) is the value of horizontal coordinate on the \( t \)-distribution corresponding to the specified degree of freedom (DF), \( \alpha \) is the level of confidence interval, and \( V_e \) is the variance of error of the predicted response. The \( \Delta Ra \) value for SR is calculated using the values of error variance from Table 3. The value of \( \alpha \) is taken as 0.05. The value of \( \Delta Ra \) for SR models is calculated as 0.307 \( \mu m \).

4.3. The Optimization of the EDM Process Parameters for Obtaining Better Surface Quality

Optimization of machining parameters increases the utility of electric discharge machining to a great extent. In this context, an effort has been made to estimate the optimum machining cutting conditions to produce the best possible surface quality within the constraints. SR can be minimized by setting process parameters at optimum levels. The problem of constrained optimization using a developed SR model for EDM of AlSiTi ceramic composite is formulated and is given by

\[
\min_s \{ \frac{1}{2} s^T H_s + s^T g \quad \text{such that} \quad \| D_s \| \leq \Delta \}
\]

subjected to constrained \( \| D_s \| \leq \Delta \). The length of the step is accepted if \( \| D_s \| \leq \Delta \). Here \( g \) is gradient of \( f \) (a vector of first derivatives) at the current point \( x \), \( H \) is the Hessian matrix (the symmetric matrix of second derivatives), \( D \) is a diagonal scaling matrix, \( \Delta \) is a positive scalar, and \( \| \cdot \| \) is a 2 norm.

There is no need to provide an accurate solution to Eq. (8) because it requires large computation time, which is proportional to several factorizations of \( H \). Therefore, for large-scale problems, an approximation approach is recommended, and the same is used in the present work. The approximation approach followed in the optimization is to restrict the trust-region subproblem to a two-dimensional subspace \( S \). Once the subspace \( S \) has been computed, the solution of Eq. (8) is trivial. The dominant work is therefore shifted to determination of the subspace. The two-dimensional subspace \( S \) is determined with the preconditioned conjugate gradient process described below.

4.5. Preconditioned Conjugate Gradient Method

In the conjugate gradient algorithm, the step size is adjusted after each iteration. A search is made along the conjugate gradient direction \( S \) to determine the step size \( s \), which minimizes the performance function along that line. The two-dimensional subspace \( S \) is given as \( S = (s_1, s_2) \), where \( s_1 \) is in the direction of the gradient \( g \), and \( s_2 \) is either an approximate Newton direction, i.e., a solution to

\[ H \cdot s_2 = -g \]

or a direction of negative curvature,

\[ s^T \cdot H \cdot s_2 < 0. \]

The philosophy behind this choice of \( S \) is to force global convergence (via the steepest descent direction or negative

\[ \ln(\Delta ra) \]
DETERMINATION OF AN OPTIMUM PARAMETRIC COMBINATION 681

2. Solve Eq. (7) to determine the trial step \( x \);
3. If \( f(x + s) < f(x) \), then \( x = x + s \);
4. Adjust \( \Delta \).

These four steps are repeated until the convergence is achieved (Hessian matrix \( H \) should be positive semidefinite).

A standard function of MATLAB 7.0, namely, “fmincon” that can handle a large-scale optimization problem with nonlinear equality as well as inequality constraint is used for the purpose. The obtained process parameters, which give minimum SR, are presented in Table 4. EDM is carried out on the AlSiTi workpiece using a copper electrode having 8.5 mm diameter at optimum set of process parameters and SR is measured. Measured value is compared with the value obtained from the SR model as given in Table 4. It can be seen that developed models can predict the SR accurately within 95% confidence interval.

This optimization methodology can be used to determine minimum SR with given constraints and also identifies the conditions at which EDM operation has to be carried out in order to get the better surface finish. The application of a trust region method to obtain optimal machining condition in EDM of ceramic composite will be quite useful at the computer-aided process planning (CAPP) stage in the production of electric discharge machined parts as such data is not available for EDM of ceramic composites.

5. Conclusions
1. The two-stage effort of obtaining a SR model by RSM and optimization of this model by a trust region method have resulted in a fairly useful method of obtaining process parameters in order to attain the improved surface quality.
2. The investigation indicates that parameters discharge current, pulse-on time, and duty cycle are the primary factors influencing the SR of AlSiTi ceramic composite during EDM.
3. Pulse-on time is found to be the dominant parameter influencing SR.
4. It was also observed that an increase in discharge current increases the SR. The confirmation test showed that developed models can predict the SR accurately within 95% confidence interval.
5. The methodology adopted establishes the optimization of AlSiTi ceramic composite in EDM and hence facilitates the effective use of EDM machinable ceramic composite in industrial applications by reducing the cost of machining.

ACKNOWLEDGMENTS
The authors wish to thank Mr. John J. Schuldies, President, Industrial Ceramic Technology Inc., Ann Arbor Michigan, USA, for providing help and support in procurement of the workpiece material for the research work.

References


