

# Heat Transfer Analysis of 2-Stage G-M Cryocooler at 4 K Steady State Temperature for the Determination of its Capacity

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**Abstract**— A two stage Gifford-Mc Mahon Cryocoolers are used to produce ultra-low temperatures in the range of 4 K to 40 K in the cryogenics experimentations. Commercially available G-M Cryocoolers have wide range of cooling capacity. Heat transfer analysis has been carried out in the present study on the G-M Cryocooler at no load condition by considering experimental observations as input data. The rated capacity of the G-M Cryocooler is 1.5 W at 4 K temperature. By producing 4 K temperature on the cryocooler at no load condition, the same has been analysed by heat transfer approach. All the modes of heat transfer has been calculated at steady temperature of 4 K with steady vacuum level in the range of  $10^{-5}$  Torr. The calculated capacity of the Cryocooler has been found near to the rated capacity of 1.5 W at 4 K. The same approach can be made useful to calculate the capacity of ultra-low temperature refrigeration systems.

**Index Terms**— Gifford-Mc Mahon Cryocooler, Heat transfer, Radiation Shield, Turbo Molecular Pump, Gas conduction

## 1 INTRODUCTION

Gifford-Mc Mahon cryocooler basically works on G-M cycle which is a closed system containing helium as a working fluid. The function of the cold head is to produce continuous closed cycle refrigeration at temperatures, depending upon the heat load imposed, in the range of 25 k to 40 k for the first stage and in the range of 3.5 K to 4.5 K for the second stage cold station. The cold head has three major components, the drive unit, the cylinder and the displacer- regenerator assembly, which is located inside the cylinder. With newly developed rare earth regenerator material and with very unique structure the model RDK-415 cold head has its second stage refrigeration capacity of 1.5 watt at 4.0 K. Functionally, the high pressure helium gas from the compressor unit will be supplied to the cold head through the helium gas supply connector. The supply gas will be passed into the displacer- regenerator assembly to the crank case through the motor housing, and finally will be returned to the compressor unit through the helium gas return connector. The helium gas expansion in the displacer-regenerator assembly will be provided cooling condition for the first and second stage cold stations. The model of the G-M cycle cryocooler cold head is shown in figure 1. Cold head consists of a cylinder, displacer no. 1, displacer no. 2, drive mechanism and cold head drive motor. No1 displacer is connected to the Scotch Yoke mechanism which can be driven by the cold head drive motor through the crank with bush so that the rotation of the cold head drive motor can be varied to reciprocating motion of

Scotch Yoke and displacers.

The Rotary valve system is furnished to control the helium gas intake and exhaust timing. The rotary valve is also coupled to the cold head drive motor through crank, so intake and exhaust operation is synchronized with the position of the displacer. The displacer is a loose fit in the cylinder except at the top and where it is equipped with a dynamic (sliding) seal to prevent leakage passed through the clearance between the displacer and cylinder. The displacers consist of regenerator material which cools the gas when passing downwards to the cold space. Rare earth regenerator material is used in the second stage displacer to produce the cooling capacity at the temperature of 4.0 K. The pressure above and below the displacer is the same except for small pressure drops across the regenerator when gas is flowing through it. Virtually no physical work is required to move the displacer in the cylinder. No work is done on the gas and the gas won't do any work on the displacer. The pressure in the system is increased or decreased by operation of the inlet or outlet valves.

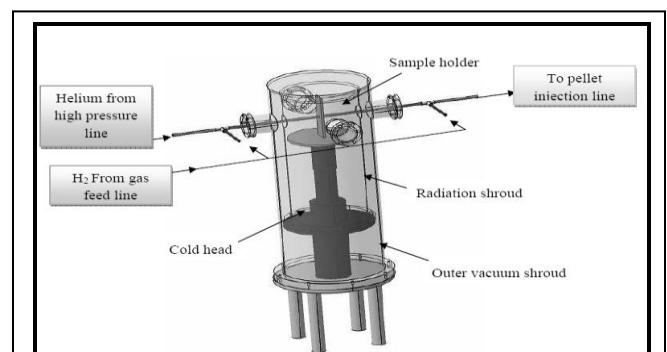


Fig. 1. Cold head and vacuum vessel assembly for the G-M cryocooler

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## 2 HEAT TRANSFER ANALYSIS AT STEADY STATE

### 2.1 Important Parameters

Heat transfer analysis in this paper has been carried out by considering 4 K steady temperature of cold finger. The atmospheric temperature is considered as 300 K. The actual set-up has been used for freezing the hydrogen by allowing it to enter in the cryocooler through an SS pipe so the heat transfer through pipe is considered as conduction load. The cold head is covered with vacuum shroud which is been evacuated by turbomolecular vacuum pumping system. There is one radiation shield between the outer vacuum vessel and cold head. All three modes of heat transfer have been calculated to find out the capacity of cryocooler. Due to high vacuum level in the cooler, convection heat transfer is replaced by molecular gas conduction. The level of vacuum observed during the experimentations was  $5.64 \times 10^{-4}$  Pa.

### 2.2 Radiation through One Radiation Shield of Aluminum

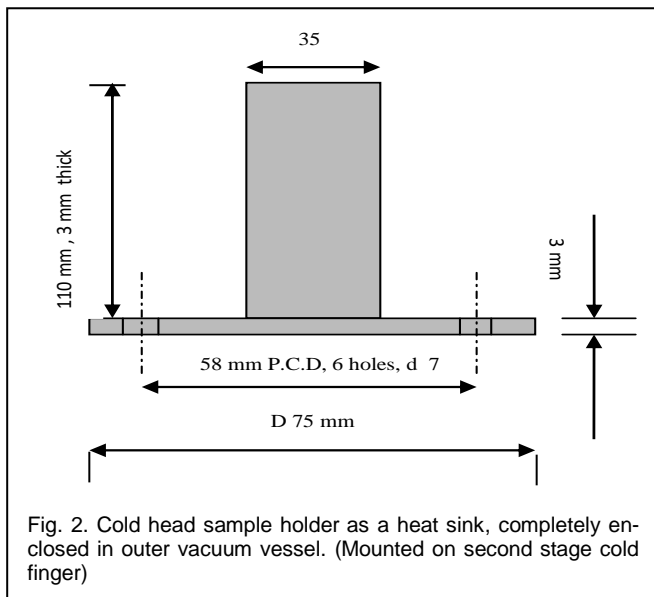


Figure 2 shows the dimensions of sample holder. Sample holder is mounted on the cold head of the second stage of the cryocooler. Temperature at the bottom surface of the sample holder is observed 4 K. The average temperature of entire sample holder is assumed to be 10 K. To calculate the radiation heat transfer rate from atmosphere to sample holder emissivity factor ( $F_e$ ) can be calculated from the following relation.

$$\frac{1}{F_e} = \left[ \frac{1}{e_1} + \frac{A_1}{A_S} \left\{ \frac{1}{e_S} - 1 \right\} + \frac{A_1}{A_S} \left\{ \frac{1}{e_S} + \frac{A_S}{A_2} \left( \frac{1}{e_2} - 1 \right) \right\} \right] \quad (1)$$

$F_e$  = emissivity factor

$e_1$  = emissivity of cold surface (sample holder) = 0.06 for copper

$e_2$  = emissivity of hot surface (vacuum shroud) = 0.16 for S.S. 316

$e_S$  = emissivity of radiation shield = 0.05 for aluminum

$A_S$  = area of radiation shield surface = 0.267 m<sup>2</sup>

$A_1$  = area of sample holder = 0.0154 m<sup>2</sup>

$A_2$  = area of outer vacuum shroud = 0.496 m<sup>2</sup>

From the equation (1), the value of  $F_e$  = 0.0524

Now, Radiation heat transfer rate can be obtained by following relation

$$Q_R = F_e F_{1-2} \sigma A_1 (T_2^4 - T_1^4) \quad (2)$$

Where,

$F_{1-2}$  = Configuration factor = 1 (completely enclosed)

$\sigma$  = Steffen-Boltzman Constant =  $5.67 \times 10^{-8}$  W/m<sup>2</sup> K<sup>4</sup>

$T_2$  and  $T_1$  = Outer and Inner temperatures respectively  
= 300 K and 10 K

### 2.3 Residual Gas Molecular Conduction

When the mean free path of the gas molecules is small enough compared to the geometrical dimensions, the heat transfer occurs in the continuum region and convective heat transfer can be considered. Due to high vacuum level in the cryocooler configuration, the heat transfer is due to free molecular region. To calculate the same, following relations can be used.

$$Q_g = G p A_1 (T_2 - T_1) \quad (3)$$

Where,

$$G = \frac{\gamma+1}{\gamma-1} \left[ \frac{g_c R}{8\pi T} \right]^{\frac{1}{2}} F_a \quad (4)$$

Where,

R = Gas Constant

T = Temperature of outer vessel

$g_c$  = Conversion factor = 1 in SI system

$\gamma$  = 1.4 for air

$F_a$  = Accommodation factor

$$\frac{1}{F_a} = \frac{1}{a_1} + \frac{A_2}{A_1} \left[ \frac{1}{a_2} - 1 \right] \quad (5)$$

$a_1$  = accommodation co-efficient of sample holder = 1

$a_2$  = accommodation co-efficient of vacuum vessel = 0.8

$F_a$  = 0.9923

When the pressure inside the system is too low, the continuum region of convection shifts to free molecular gas conduction. As per the Knudsen number equation, if mean free path of molecules is high enough than the geometrical dimensions then gaseous conduction comes into the picture.

Mean free path

$$\lambda = \frac{\mu}{p} \times \left[ \frac{\pi RT}{2g_c} \right]^{\frac{1}{2}}$$

Where,

$\mu$  = Gas viscosity at temperature T =  $18.47 \times 10^{-6}$  Pa.s

p = Residual gas pressure =  $5.64 \times 10^{-4}$  Pa (Experimental)

R = Gas constant = 287 J/kg K for air

T = Absolute Temperature = 300 K

So  $\lambda$  = 12.02 m, which very high compared to the geometrical dimensions of the cryocooler set up. So the convection is replaced by the molecular conduction heat transfer.

By using equation (3), heat transfer rate by residual gas conduction can be obtained.

## 2.4 Conduction through SS Pipe from Atmosphere to Sample Holder

The original cryocooler set up has been used for freezing the hydrogen gas in the sample holder. To transfer the hydrogen gas into the cooler, stainless steel pipes have been used. The only solid contact between the outside atmosphere and inside sample holder is through pipes, which imparts conduction heat load to the cooler. In this section, the conduction heat transfer rate through transfer pipes has been calculated. In the cryogenic temperature region, the thermal conductivity of the material can not be taken as constant as the variation in the conductivity value is considerable with respect to temperature. As a result, to calculate the conduction heat transfer from the pipes, thermal conductivity integral and conduction shape factor should be found first. Two small pipes of 100 mm and 1.175 mm thickness have been inserted into the cryocooler from both the openings.

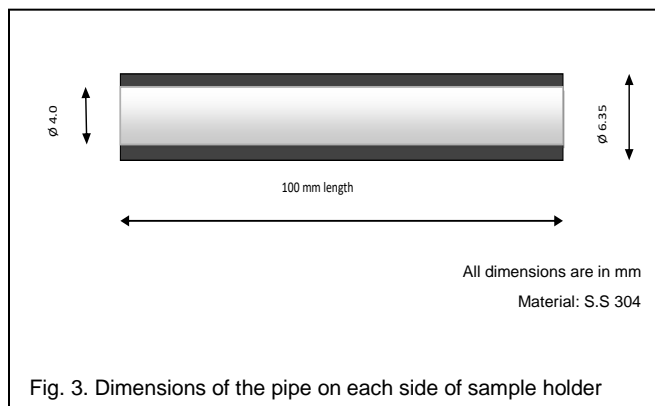


Fig. 3. Dimensions of the pipe on each side of sample holder

Two sensors have been mounted on the pipe on both the ends shows the temperatures for conduction heat transfer.

$$T_1 = 300 \text{ K}$$

$$T_2 = 60 \text{ K}$$

$$\text{Conduction Area } A = 19.1 \text{ mm}^2$$

$$\text{Conduction Shape Factor } S = A/L = 0.191 \text{ mm}$$

Thermal Conductivity Integral

$$K_1 = 3060 \text{ W/m at } 300 \text{ K}$$

$$K_2 = 198 \text{ W/m at } 60 \text{ K}$$

Total heat transfer rate from both the pipes in the form of conduction can be now obtained from the following relation

$$Q = 2 \times S (K_1 - K_2) \quad (6)$$

## 3 CONCLUSION

The amount of radiation heat transfer rate obtained from the equation (2) is 0.371 W. Heat transfer rate by molecular gas conduction obtained by equation (3) is 0.00292 W and heat transfer rate from pipe conduction obtained from equation (6) is 1.0932 W. All these heat transfer rate values have been obtained at constant temperature of 4 K of cold finger on which the sample holder is mounted. The total amount of heat transfer rate at 4 K constant temperature is  $0.371 + 0.00292 + 1.0932 = 1.46712 \text{ W}$ . The rated capacity of the cryocooler is 1.5 W at 4

K temperature. The rated capacity and the calculated capacity of the Cryocooler are having near values. Although in the calculations, the transfer of heat through temperature sensor wires haven't been considered. This Methodology can be made useful to find the capacity of ultra low temperature refrigerators even after the years of service when the performance of cooler deteriorates.

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## REFERENCES

- [1] <http://www.janis.com/Products/productoverview/4KCryocooler/RDK-415DClosedCycleRefrigerator.aspx#>
- [2] M.N.irmanus. Janis Research Company "Introduction to Laboratory Cryogenics"
- [3] R.F.Barron, Cryogenic System/Book
- [4] R.F.Barron, Cryogenic Heat transfer/Book
- [5] R.G.Ross.Jr, D.L.Johnson, "Nasa's advanced Cryocooler Technology" Cryogenic Engineering Conference, Keystone, Colorado, September 2005
- [6] R.H. Kropshot, Corruccini "Selected total minimum emissivities", 1959, 1963
- [7] Partington, "Summarized literature on accommodation co-efficients", 1952